



Cambridge International AS & A Level

CANDIDATE
NAME

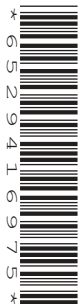
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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 (a) Find a and b such that

$$z^8 - iz^5 - z^3 + i = (z^5 - a)(z^3 - b). \quad [1]$$

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(b) Hence find the roots of

$$z^8 - iz^5 - z^3 + i = 0,$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [6]

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- (b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{r}{2r^2-1}$. [3]
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- 4 By considering the binomial expansions of $\left(z + \frac{1}{z}\right)^5$ and $\left(z - \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\tan^5 \theta = \frac{\sin 5\theta - a \sin 3\theta + b \sin \theta}{\cos 5\theta + a \cos 3\theta + b \cos \theta},$$

where a and b are integers to be determined.

[7]

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5 The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - 3y = 4e^{-x}.$$

(a) Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the differential equation. [4]

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(b) Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. [6]

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- 6 (a) Starting from the definitions of \sinh and \cosh in terms of exponentials, prove that

$$2 \sinh^2 x = \cosh 2x - 1. \quad [3]$$

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- (b) Find the solution to the differential equation

$$\frac{dy}{dx} + y \coth x = 4 \sinh x$$

for which $y = 1$ when $x = \ln 3$. [7]

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7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{3}{2}} (4+x^2)^{-\frac{1}{2}n} dx$.

(a) Find the exact value of I_1 , expressing your answer in logarithmic form. [3]

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(b) By considering $\frac{d}{dx}(x(4+x^2)^{-\frac{1}{2}n})$, or otherwise, show that

$$4nI_{n+2} = \frac{3}{2}\left(\frac{2}{5}\right)^n + (n-1)I_n. \quad [5]$$

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(c) Find the value of I_5 . [3]

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- 8 (a) Find the value of a for which the system of equations

$$\begin{aligned} 13x + 18y - 28z &= 0, \\ -4x - ay + 8z &= 0, \\ 2x + 6y - 5z &= 0, \end{aligned}$$

does not have a unique solution. [2]

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The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$$

- (b) Find the eigenvalue of \mathbf{A} corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. [1]

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- (c) Find a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [8]

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